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Stochastic Control and Dynamic Optimisation

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Abstract

Dynamic optimisation and stochastic control encompass several problems central to control engineering.

- Dynamic optimisation addresses the question of how to design control laws to optimise one or more performance criteria. In this course two classes of dynamic optimisation problems will be considered, namely (single-objective) optimal control and (multi-objective) nonzero-sum differential games. Obtaining solutions of such problems poses, in general, an intractable problem. Thus, in addition to a review of the classical theory, strategies to systematically construct approximate solutions – at a relatively low computational expense – will be presented.
- Control of stochastic differential equations (SDEs) has been successfully used in a variety of theoretical and applied scientific fields, such as system biology and finance. One way to view SDEs is to interpret the stochastic processes in the equations as a means to model uncertainty. In this sense, stochastic systems offer a powerful modelling framework for engineering applications. The course will provide an introduction to stochastic control.

The course will initially develop these topics in two independent units which will be merged towards the end of the course with topics such as stochastic optimal control.

Course outline

• Dynamic optimisation

Considering optimal control and differential games, we will see how their solutions can be obtained by means of either the Dynamic Programming approach or Pontryagin's minimum principle. Recognising the computational burden associated with solving such problems exactly, we will consider various strategies to solve optimal control problems and differential games approximately at (significantly) reduced computational cost. Practical examples will be provided alongside the theoretical considerations.

• Introduction to Stochastic Control

This unit will assume no prior knowledge of stochastic control and SDEs. The aim of the unit is to develop a working knowledge of control of SDEs. We will start by quickly introducing some elements of measure theory and probability which are instrumental for the development of the course. We will then construct the Brownian motion, discuss its relation with white noise, cover the notion of stochastic integral and other topics related to SDEs, such as existence and uniqueness of the solution, linear SDEs and simulation. We will then cover a selection of advanced topics: various concept of stability and elements of optimal control (connecting with the other unit of the course).