Mesoscopic Hybrid Automaton for Longitudinal Vehicle Control

A. Iovine, D. Bianchi, E. De Santis, M.D. Di Benedetto

***Abstract*— In this paper a mesoscopic hybrid automaton is introduced in order to obtain an human-behavior based vehicle controller. A microscopic hybrid automaton model for longitudinal vehicle control based on human psycho-physical behavior is first presented. Advantages of using hybrid automaton are that all possible driver behavior depending on next vehicle are considered. Then a rule for changing time headway depending on macroscopic quantities is used to describe importance of interaction among all next vehicles and their impact on driver performance. The resulting mesoscopic vehicle model has a set of possible behaviors that contains more real-life situations than the purely microscopic one.**

I. INTRODUCTION

Nowadays traffic control is one of the most studied problem in engineering. This is due to its high impact in human life: progressing the knowledge and control over traffic systems means to raise life quality [1]. The main purpose of traffic control is to improve the traffic management depending on a variety of different goals: congestion, emissions and travel time reduction, safety increments etc....

Over the time a multitude of traffic control systems have been generated (see [2], [3], [4], [5]). A classification can be done between macroscopic and microscopic models: in the first ones traffic is represented as a continuum flow and its values are described by mean values variables (see [6], [7]), while the other ones describe each single vehicle on the road and their interaction (see [1]). Both models have advantages but also drawbacks depending on their exclusive aggregation type. In order to avoid drawbacks scientists are now focusing on mesoscopic models, a new kind of models which combine microscopic and macroscopic approaches in which parameters of the microscopic model depend on macroscopic quantities. Another classification distinguishes centralized control strategies from decentralized ones: respectively, there will be a single control strategy that has been taken by a single controller, which knows everything, or a set of controllers, each one with control strategy based on its partial knowledge. Centralized control strategies require an appropriate infrastructure, which is usually suitable, while basis for decentralized ones depend on single agents connectivity. Thanks to new telecommunication results, communication among vehicles is now possible without sophisticated equipment installation (see [8], [9]).

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In this paper we focus on a decentralized mesoscopic control approach. An hybrid system is chosen for representing vehicle dynamics because of its ability to include in the same mathematical model various different human behaviors. The vehicle is modeled as an agent controlled by an hybrid automaton that represents the needed driver actions for obtaining the desired dynamics, which will be influenced by different conditions-dependent ways of proceeding. Agents share information regarding their state among them. Switching through different control strategies the driver can achieve his or her task on the basis of the environmental information.

Hybrid system allows us to consider the wide range of different behavior a driver could have. We used two different types of microscopic models: stymulus-response and psycho-physical. The former models type describes vehicle continuous dynamics depending on the stymulus (i.e. the continuous dynamics) of the ahead vehicle [2]. The latter models type uses thresholds for deciding which continuous dynamics the vehicle will use [10].

In real life microscopic model parameters are related to macroscopic quantities, such as traffic density. Being variance a density-dependent function [11], a variance-driven adaptation mechanism is adopted for changing thresholds depending on the local mean speed value and local variance to improve the overall systems performance. To quantify controller performance improvements we compare cases taking into account quantities such as safety distance, fuel consumption and emission rate.

The paper is organized in 5 sections. In Section II the microscopic hybrid automaton of a single vehicle will be described, arguing that this mathematical model is the most appropriate tool. Then in Section III a variance-driven time headway mechanism will be introduced and used for the hybrid automaton; by adding this mechanism, which depends on macroscopic quantities, to the previously defined microscopic automaton, the resulting hybrid automaton can be defined mesoscopic. Section IV provides simulation results about automaton behavior. Summary and conclusions are outlined in Section V.

II. HYBRID MODEL OF MICROSCOPIC TRAFFIC MODEL

Microscopic models describe traffic flow dynamics in terms of single vehicles. In this section, a microscopic traffic model based on classical Car-Following literature models is introduced. The considered situation concerns a single lane road case. The leading car, called the "leader", is represented by the number n, while the later ones, called "followers" are represented with numbers $n+1$, $n+2$,... (cf. Figure 1). For

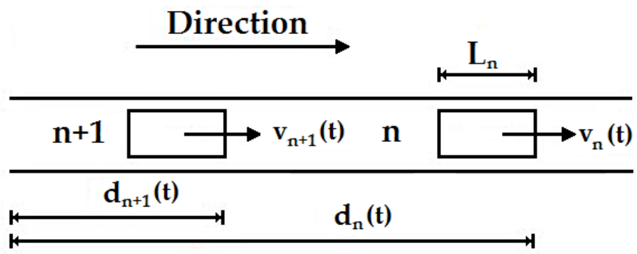

Fig. 1. n is the leader vehicle and $n+1$ the follower one. L_n is the vehicle length (all vehicles are supposed to be identical) and d_n, d_{n+1} are the covered distances by vehicles n and $n+1$ respectively.

now, let us assume that each leader shares information about its position and speed with its follower.

According to hybrid automaton definition given in [12], the hybrid system will be the tuple

$$\mathcal{H} = (Q, X, f, Init, Dom, \mathcal{E}, \mathcal{G}, \mathcal{R}) \tag{1}$$

where:

- Q is a finite set of N discrete states;
- $X \subseteq \mathbb{R}^n$ is the continuous state space;
- $f(\cdot,\cdot) : Q \times X \to \mathbb{R}^n$ is a vector field that associates to each discrete state $q \in Q$ the continuous time-invariant dynamics
$$\dot{x} = f_q(x, u), \tag{2}$$
and the output map $y = g_q(x)$. Given an initial condition x_0 at time t_0 and a control input $u|_{t_0}^t : [t_0, t]$, we denote the solution at time t according to f_q by
$$x(t) = x_q(t, x_0, u|_{t_0}^t).$$
The solution of the above differential equation exists and it is unique, provided that f_q is Lipschitz continuous with respect to its arguments;
- $Init \subseteq Q \times X$ is the set of initial discrete and continuous conditions;
- $Dom(\cdot) : Q \to 2^X$ is a domain;
- $\mathcal{E} \subseteq Q \times Q$ is a set of edges;
- $\mathcal{G} : \mathcal{E} \to 2^X$ is a map associating to each transition $e \in \mathcal{E}$ a set $\mathcal{G}(e)$ called guard;
- $\mathcal{R}(\cdot,\cdot) : \mathcal{E} \times X \to 2^X$ is a reset map.

A. Continuous states

It is assumed that the vehicles motion is an accelerated-decelerated one; we denote the traveled distance as $d(t)$. Representing space as x_1 and speed as x_2, the vector state space will be

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ u(t, x_1(t), x_2(t)) - f_r \end{bmatrix} \tag{3}$$

where $u(t, x_1(t), x_2(t))$ is the acceleration/deceleration, that will be shown depending on space and speed, and f_r is the friction term. Let us consider bounds for $u(t, x_1(t), x_2(t))$ such that $u_{min} \leq u(t, x_1(t), x_2(t)) \leq u_{max}$, with the maximum deceleration $u_{min} < 0$ and the maximum acceleration $u_{max} > 0$.

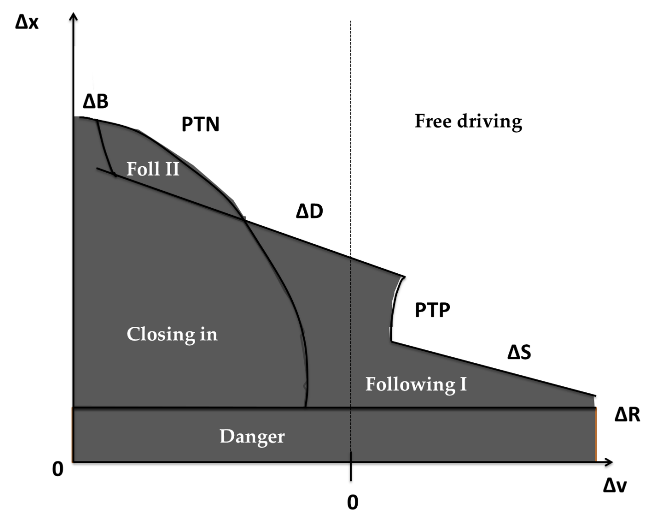

Fig. 2. The different thresholds and regimes in Fritzsche car-following model. There are two main areas; one is related to the so-called "leader phase" (Free-driving) where the vehicle does not take into account its ahead vehicle, and the other one related to the "follower phase" (Following I, Following II, Closing in, Danger) where vehicle dynamic depends on its ahead vehicle.

In the considered case there will be $X = \mathbb{R}^2$. We will define $\Delta d(t) = d_n(t) - d_{n+1}(t)$ as the distance and $\Delta v(t) = v_n(t) - v_{n+1}(t)$ as the speed difference between vehicle n and $n+1$ (see Figure 1). So the equations for $\Delta d(t)$ and $\Delta v(t)$ become $\Delta x_1(t) = x_{1_n}(t) - x_{1_{n+1}}(t)$ and $\Delta x_2(t) = x_{2_n}(t) - x_{2_{n+1}}(t)$. In the following the time dependence will be omitted for sake of simplicity.

B. Discrete states

For setting discrete states of the hybrid automaton the psycho-physical Car-Following model by Fritzsche (see [10]) is used. The Fritzsche model accounts for human perception in the definitions of the model regimes. In the model the distance-relative velocity plane of the leader and follower vehicles is considered. The plane is divided in different regions; in each one of them, the driver will act differently. The model represents in $\Delta v - \Delta d$ $(\Delta x_2 - \Delta x_1)$ plane two thresholds for perception of speed differences (PTN, negative, and PTP, positive) and four thresholds for the followers space headway to its leader (the risky distance ΔR, the safe distance ΔS, the desired distance ΔD and the braking distance ΔB) (see Figure 2). The thresholds become

$$PTN = -k_{PTN} \cdot (\Delta x_1 - s_n)^2 - f_x \tag{4}$$

$$PTP = k_{PTP} \cdot (\Delta x_1 - s_n)^2 + f_x \tag{5}$$

$$\Delta R = s_n + T_r x_{2_n} \tag{6}$$

$$\Delta S = s_n + T_s x_{2_{n+1}} \tag{7}$$

$$\Delta D = s_n + T_d x_{2_{n+1}} \tag{8}$$

$$\Delta B = \Delta R + \frac{(\Delta x_2)^2}{|b_{min}| + a_n^-} \tag{9}$$

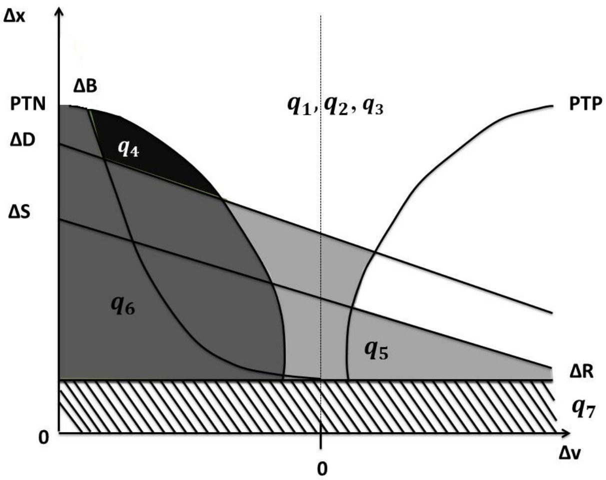

Fig. 3. Automaton discrete states corresponding to different thresholds and regimes in Fritzsche car-following model.

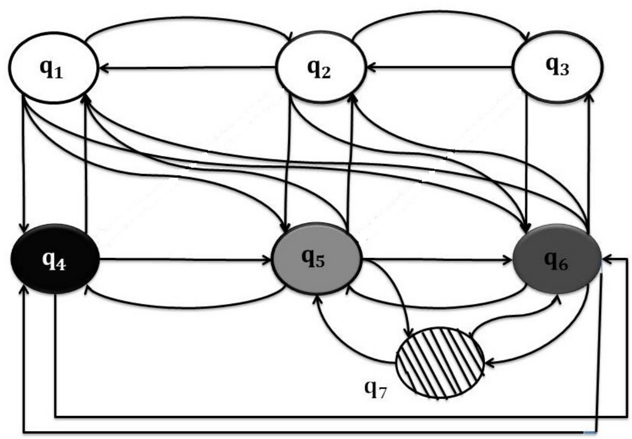

Fig. 4. The obtained hybrid automaton: discrete states colors are related to Fritzsche's model area divisions.

where k_{PTN}, k_{PTP}, f_x, b_{min}, a_n^- are model parameters, s_n is the effective desired length from the ahead vehicle and T_r, T_s, T_d are time headways. We define an hybrid model with 7 discrete states according to Fritzsche cars interaction spaces definitions (see Figure 3)

$$Q = (q_1, q_2, q_3, q_4, q_5, q_6, q_7) \tag{10}$$

defined as follow:

1) In the "Free driving" region the vehicle can run freely because the leader vehicle is either too far away or faster or both. The follower is "free", and he acts as a his own "leader". There will be three possibilities of acting, described by three distinguished discrete states:
   a) q_1: the driver is supposed to accelerate, because his speed is less than the desired one.

$$f_{q_1}(X) = \begin{bmatrix} x_{2_{n+1}} \\ \alpha_1^+ \cdot (v_{des} - x_{2_{n+1}}) - f_r \end{bmatrix} \tag{11}$$

   b) q_2: the driver does not make any action, because the speed is more or less equal to the desired one.

$$f_{q_2}(X) = \begin{bmatrix} x_{2_{n+1}} \\ -f_r \end{bmatrix} \tag{12}$$

   c) q_3: the driver is supposed to decelerate, because his speed is higher than the desired one.

$$f_{q_3}(X) = \begin{bmatrix} x_{2_{n+1}} \\ \alpha_3^- \cdot (v_{des} - x_{2_{n+1}}) - f_r \end{bmatrix} \tag{13}$$

2) q_4: in the "Following II" region the driver has noticed that he is closing in on the vehicle in front but the space headway is too large, so he keeps accelerating but with different parameters; this time acceleration will depend on relative speed and distance, following the model in [3].

$$f_{q_4}(X) = \begin{bmatrix} x_{2_{n+1}} \\ \alpha_4^- \cdot \frac{\Delta x_2}{\Delta x_1} x_{2_{n+1}} - f_r \end{bmatrix} \tag{14}$$

3) q_5: in the "Following I" region speed difference and distance are small, so the driver takes no conscious action; there will be no acceleration or deceleration in this discrete state.

$$f_{q_5}(X) = \begin{bmatrix} x_{2_{n+1}} \\ -f_r \end{bmatrix} \tag{15}$$

4) q_6: in the "Closing in" region the speed difference is large and the distance is not, so the driver has to decelerate; he will do it depending on distance and relative speed, according to the model in [13].

$$f_{q_6}(X) = \begin{bmatrix} x_{2_{n+1}} \\ \frac{x_{2_n}^2 - x_{2_{n+1}}^2}{2(\Delta x_1 + \Delta R + x_{2_n} \cdot \Delta t)} - f_r \end{bmatrix} \tag{16}$$

5) q_7: in the "Danger" region the distance from the leading vehicle is very small and the driver uses his maximum deceleration.

$$f_{q_7}(X) = \begin{bmatrix} x_{2_{n+1}}(t) \\ u_{min} - f_r \end{bmatrix} \tag{17}$$

Here α_1^+, α_3^- and α_4^- are sensitivity parameters, v_{des} is the desired speed the driver wants to achieve, Δt is the time instant (the simulation time).

C. Domains, edges and guard conditions

Discrete state domains come from the speed-position plane using logic conditions and thresholds. Also edges set $\mathcal{E}$ can be defined in the same way.

$$\begin{aligned}\mathcal{E} = \{&(q_1, q_2), (q_1, q_4), (q_1, q_5), (q_1, q_6), \\ &(q_2, q_1), (q_2, q_4), (q_2, q_5), (q_2, q_6), \\ &(q_3, q_2), (q_3, q_5), (q_3, q_6), (q_4, q_1), \\ &(q_4, q_5), (q_4, q_6), (q_5, q_1), (q_5, q_2), \\ &(q_5, q_4), (q_5, q_6), (q_5, q_7), (q_6, q_1), \\ &(q_6, q_2), (q_6, q_3), (q_6, q_4), (q_6, q_5) \\ &(q_6, q_7), (q_7, q_5), (q_7, q_6)\}\end{aligned} \tag{18}$$

In general the domain of discrete state q_i is

$$Dom(q_i) = z_i(x_{2_{n+1}}, v_{des}, \Delta x_1, \Delta x_2, PTP, PTN, \Delta R, \Delta D, \Delta S, \Delta B) \tag{19}$$

with $i \in [1, 7]$, where z_i is a function that maps X in q_i depending on its own position related to the defined thresholds and its own speed related to the desired one (see Figures 3 and 4).

$$Dom(q_1) = \{x \in \Re^2 : (0 \le x_{2_{n+1}} < v_{des}) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]\} \tag{20}$$

$$Dom(q_2) = \{x \in \Re^2 : (v_{des} \le x_{2_{n+1}} < v_{des} + \epsilon) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]\} \tag{21}$$

$$Dom(q_3) = \{x \in \Re^2 : (x_{2_{n+1}} \ge v_{des} + \epsilon) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D)) \tag{22}$$

$$Dom(q_4) = \{x \in \Re^2 : (0 \le x_{2_{n+1}} < v_{des} + \epsilon) \wedge (\Delta x_2 \le PTN) \wedge (\Delta x_1 > \Delta D) \wedge (\Delta x_1 \ge \Delta B)\} \tag{23}$$

$$Dom(q_5) = \{x \in \Re^2 : [(\Delta x_2 > PTN) \wedge (\Delta x_2 \le PTP) \wedge (\Delta x_1 \le \Delta D) \wedge (\Delta x_1 > \Delta R)] \vee [(\Delta x_2 > PTP) \wedge (\Delta x_1 \le \Delta S) \wedge (\Delta x_1 > \Delta R)]\} \tag{24}$$

$$Dom(q_6) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge (\Delta x_2 \le PTN) \wedge [(\Delta x_1 < \Delta B) \vee (\Delta x_1 < \Delta D)]\} \tag{25}$$

$$Dom(q_7) = \{x \in \Re^2 : (\Delta x_1 \le \Delta R)\} \tag{26}$$

The same reasoning used for domains definition will be adopted for guard conditions sets. Furthermore, we will set them to be mutually exclusive:

$$\mathcal{G}(q_i, q_j) = z_{ij}(x_{2_{n+1}}, v_{des}, \Delta x_1, \Delta x_2, PTP, PTN, \Delta R, \Delta D, \Delta S, \Delta B) \quad \forall (i, j) : e_{ij} \in \mathcal{E} \tag{27}$$

with $i, j \in [1, 7]$, where z_{ij} is a map that associates the set $\mathcal{G}(q_i, q_j)$ to the transition e_{ij} for every (i, j) such that $e_{ij} \in \mathcal{E}$.

$$\mathcal{G}(q_1, q_2) = \{x \in \Re^2 : [(x_{2_{n+1}} > v_{des}) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]]\} \tag{28}$$

$$\mathcal{G}(q_1, q_4) = \{x \in \Re^2 : [(\Delta x_1 > \Delta D) \wedge (\Delta x_1 > \Delta B) \wedge (\Delta x_2 \le PTN)]\} \tag{29}$$

$$\mathcal{G}(q_1, q_5) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge [((\Delta x_2 < PTP) \wedge (\Delta x_1 \le \Delta D) \wedge (\Delta x_2 \ge PTN)) \vee ((\Delta x_1 < \Delta S) \wedge (\Delta x_2 > PTP))]\} \tag{30}$$

$$\mathcal{G}(q_1, q_6) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge (\Delta x_1 \le \Delta B) \wedge (\Delta x_2 \le PTN)\} \tag{31}$$

$$\mathcal{G}(q_2, q_1) = \{x \in \Re^2 : (x_{2_{n+1}} < v_{des}) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]\} \tag{32}$$

$$\mathcal{G}(q_2, q_3) = \{x \in \Re^2 : [(x_{2_{n+1}} \ge v_{des} + \epsilon) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]\} \tag{33}$$

$$\mathcal{G}(q_2, q_4) = \{x \in \Re^2 : [(\Delta x_1 > \Delta D) \wedge (\Delta x_1 > \Delta B) \wedge (\Delta x_2 \le PTN)]\} \tag{34}$$

$$\mathcal{G}(q_2, q_5) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge [((\Delta x_2 < PTP) \wedge (\Delta x_1 \le \Delta D) \wedge (\Delta x_2 \ge PTN)) \vee ((\Delta x_1 < \Delta S) \wedge (\Delta x_2 > PTP))]\} \tag{35}$$

$$\mathcal{G}(q_2, q_6) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge (\Delta x_1 \le \Delta B) \wedge (\Delta x_2 \le PTN)\} \tag{36}$$

$$\mathcal{G}(q_3, q_2) = \{x \in \Re^2 : [(x_{2_{n+1}} < v_{des} + \epsilon) \wedge [((\Delta x_2 > PTP) \wedge (\Delta x_1 > \Delta S)) \vee ((\Delta x_2 > PTN) \wedge (\Delta x_1 > \Delta D))]\} \tag{37}$$

$$\mathcal{G}(q_3, q_5) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge [((\Delta x_2 < PTP) \wedge (\Delta x_1 \le \Delta D) \wedge (\Delta x_2 \ge PTN)) \vee ((\Delta x_1 < \Delta S) \wedge (\Delta x_2 > PTP))]\} \tag{38}$$

$$\mathcal{G}(q_3, q_6) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge (\Delta x_2 \le PTN) \wedge [(\Delta x_1 \le \Delta B) \vee (\Delta x_1 < \Delta D)]\} \tag{39}$$

$$\mathcal{G}(q_4, q_1) = \{x \in \Re^2 : (x_{2_{n+1}} < v_{des}) \wedge (\Delta x_1 > \Delta D) \wedge (\Delta x_2 > PTN)\} \tag{40}$$

$$\mathcal{G}(q_4, q_5) = \{x \in \Re^2 : (((\Delta x_1 = \Delta D) \wedge (\Delta x_2 = PTN)\} \tag{41}$$

$$\mathcal{G}(q_4, q_6) = \{x \in \Re^2 : (\Delta x_1 > \Delta R) \wedge (\Delta x_2 < PTN) \wedge ((\Delta x_1 \le \Delta B) \vee (\Delta x_1 \le \Delta D))]\} \tag{42}$$

$$\mathcal{G}(q_5,q_1)=\{x\in\Re^2:(x_{2_{n+1}}<v_{des})\wedge[((\Delta x_1>\Delta D)\wedge (\Delta x_2>PTN)\wedge(\Delta x_2<PTP))\vee ((\Delta x_1>\Delta S)\wedge(\Delta x_2\geq PTP))]\} \tag{43}$$

$$\mathcal{G}(q_5,q_2)=\{x\in\Re^2:[(x_{2_{n+1}}\geq v_{des})\wedge[((\Delta x_1>\Delta D)\wedge (\Delta x_2>PTN)\wedge(\Delta x_2<PTP))\vee ((\Delta x_1>\Delta S)\wedge(\Delta x_2\geq PTP))]\} \tag{44}$$

$$\mathcal{G}(q_5,q_4)=\{x\in\Re^2:(\Delta x_1=\Delta D)\wedge(\Delta x_2=PTN)\} \tag{45}$$

$$\mathcal{G}(q_5,q_6)=\{x\in\Re^2:[(\Delta x_1<\Delta D)\wedge (\Delta x_2<PTN)\wedge(\Delta x_1>\Delta R)]\} \tag{46}$$

$$\mathcal{G}(q_5,q_7)=\{x\in\Re^2:(\Delta x_1\leq\Delta R)\} \tag{47}$$

$$\mathcal{G}(q_6,q_1)=\{x\in\Re^2:(x_{2_{n+1}}<v_{des})\wedge (\Delta x_1>\Delta D)\wedge(\Delta x_2>PTN)\} \tag{48}$$

$$\mathcal{G}(q_6,q_2)=\{x\in\Re^2:[(v_{des}\leq x_{2_{n+1}}<v_{des}+\epsilon)\wedge (\Delta x_1>\Delta D)\wedge(\Delta x_2>PTN)\} \tag{49}$$

$$\mathcal{G}(q_6,q_3)=\{x\in\Re^2:[(x_{2_{n+1}}\geq v_{des}+\epsilon)\wedge (\Delta x_1>\Delta D)\wedge(\Delta x_2>PTN)\} \tag{50}$$

$$\mathcal{G}(q_6,q_4)=\{x\in\Re^2:(\Delta x_2<PTN)\wedge [(\Delta x_1>\Delta B)\wedge[(\Delta x_1>\Delta D)]\} \tag{51}$$

$$\mathcal{G}(q_6,q_5)=\{x\in\Re^2:(\Delta x_1\leq\Delta D)\wedge (\Delta x_1>\Delta R)\wedge(\Delta x_2\geq PTN)\} \tag{52}$$

$$\mathcal{G}(q_6,q_7)=\{x\in\Re^2:(\Delta x_1\leq\Delta R)\} \tag{53}$$

$$\mathcal{G}(q_7,q_5)=\{x\in\Re^2:(\Delta x_1>\Delta R)\wedge(\Delta x_2\geq PTN)\} \tag{54}$$

$$\mathcal{G}(q_7,q_6)=\{x\in\Re^2:(\Delta x_1>\Delta R)\wedge(\Delta x_2<PTN)\} \tag{55}$$

For a complete description see [14].

D. Initial and reset conditions

Initial states set is the entire speed-position plane considered: starting discrete state will depend on the position of the $n+1$ vehicle and on its relative speed respect to n vehicle.

$$Init=\bigcup_{i=1}^{7}\{q_i\}\times\{Dom(q_i)\}. \tag{56}$$

There will be no reset condition:

$$\mathcal{R}(q_i,q_j,X)=X \quad \forall(i,j):e_{ij}\in\mathcal{E}. \tag{57}$$

E. Automaton properties

Theorem 1: The hybrid automaton $\mathcal{H}$ is non-blocking and deterministic.

Proof: Let us define the set of reachable states $Reach$ of a general q_i discrete state

$$Reach_{q_i}=\{(q_i,\hat{x})\in Q\times X: \exists(\tau,q,x):(q(\tau'_N),x(\tau'_N))=(q_i,\hat{x})\} \tag{58}$$

and the set of states from which continuous evolution is not possible from a generic q_i discrete state

$$Trans_{q_i}=\{(q_i,\hat{x})\in Q\times X: \forall\,\epsilon>0\ \exists\ t\in[0,\epsilon):(q_i,\hat{x}(t))\notin Dom(q_i)\}. \tag{59}$$

For the set of all reachable states $Reach$ results $Init\subseteq Reach$; since every point on the considered plane could be an initial point, $Reach$ will be as

$$Reach=\bigcup_{i=1}^{7}\{Reach_{q_i}\}=\bigcup_{i=1}^{7}\{q_i\}\times\{Dom(q_i)\}. \tag{60}$$

Also the set of states from which continuous evolution is not possible from all discrete state ($Trans$) can be written as function of introduced sets:

$$Trans=\bigcup_{i=1}^{7}\{q_i\}\times\{Trans_{q_i}\}. \tag{61}$$

Then it is possible to define the intersection set between these set as

$$Reach\cap Trans=\bigcup_{i=1}^{7}\{q_i\}\times\{Reach_{q_i}\cap Trans_{q_i}\}= =\bigcup_{i=1}^{7}\{q_i\}\times\{RT_{q_i}\} \tag{62}$$

where

$$RT_{q_i}=Reach_{q_i}\cap Trans_{q_i}. \tag{63}$$

According to Lemma 4.1 in [16] the defined set will be used for constructing non-blocking conditions. In fact it is sufficient to add an appropriate guard condition $\mathcal{G}(q_i,q_j)$ to the q_i state when $Dom(q_i)\cap RT_{q_i}\neq\emptyset$ for avoiding blocking property. We then suppose that

$$\exists\ \mathcal{G}(q_i,q_j)\ :\ (q_j,\hat{x}(t))\in Dom(q_j) \quad \forall\,q_i:Dom(q_i)\cap RT_{q_i}\neq\emptyset. \tag{64}$$

Examples of these guard conditions are (28), (32) and (29).

From Lemma 4.2 in [16] we construct conditions for determinism. Based on the guard conditions defined before, there could be more possible transitions: we ban it by using mutual exclusion among all guard transitions. So second condition of Lemma 4.2 in [16] is not violated. Furthermore the first condition too, thanks to the use of domains conditions for guard definitions. Finally the last condition is met because the reset set contains a single element. Then $\mathcal{H}$ is deterministic. ∎

As example, let us consider a partial hybrid automaton $\mathcal{H}'$ composed only by discrete states q_1, q_2: because of their staying in the same region, it is possible to take into account between their guards conditions only conditions about speed.

$$\overline{\mathcal{G}}(q_1, q_2) = \{x \in \Re^2 : x_{2_{n+1}} \geq v_{des}\} \quad (65)$$

$$\overline{\mathcal{G}}(q_2, q_1) = \{x \in \Re^2 : x_{2_{n+1}} < v_{des}\} \quad (66)$$

Let us suppose now to change $\overline{\mathcal{G}}(q_2, q_1)$ for allowing discrete transitions even when the speed is equal to the desired one, $x_{2_{n+1}} = v_{des}$:

$$\overline{\mathcal{G}}(q_1, q_2) = \{x \in \Re^2 : x_{2_{n+1}} \geq v_{des}\} \quad (67)$$

$$\overline{\mathcal{G}}(q_2, q_1) = \{x \in \Re^2 : x_{2_{n+1}} \leq v_{des}\} \quad (68)$$

Let us compute the set of reachable states $Reach$ considering only these two states

$$\begin{aligned} Reach = Init = \{q_1\} \times \{x \in \Re^2 : 0 \leq x_{2_{n+1}} \leq v_{des}\} \\ \cup \{q_2\} \times \{x \in \Re^2 : x_{2_{n+1}} \geq v_{des}\} \end{aligned} \quad (69)$$

and the set of states from which continuous evolution is not possible

$$\begin{aligned} Trans = \{q_1\} \times \{x \in \Re^2 : x_{2_{n+1}} \geq v_{des}\} \\ \cup \{q_2\} \times \{x \in \Re^2 : x_{2_{n+1}} \leq v_{des}\}. \end{aligned} \quad (70)$$

According to Non-blocking theorem (see [16]), it is possible to check that theorem conditions are not violated;

$$\begin{aligned} Reach \cap Trans = \overline{\mathcal{G}}(q_1, q_2) \cap \overline{\mathcal{G}}(q_2, q_1) = \\ \{q_1\} \times \{x \in \Re^2 : x_{2_{n+1}} = v_{des}\} \\ \cup \{q_2\} \times \{x \in \Re^2 : x_{2_{n+1}} = v_{des}\} \end{aligned} \quad (71)$$

So the automaton is Non-blocking. Furthermore, it is also deterministic (see [16]) because:

1) there are no ambiguities between discrete and continuos state evolution;
2) discrete transitions have a unique destination;
3) there are no reset conditions.

So it accepts a unique infinite execution for each initial state. Hence hybrid automaton is build such that it is non blocking and deterministic.

F. Computation of equilibrium points and region of attraction

Let us consider the system

$$\begin{cases} \Delta\dot{x}_1(t) = \dot{x}_{1_n}(t) - \dot{x}_{1_{n+1}}(t) = \Delta x_2 \\ \Delta\dot{x}_2(t) = \dot{x}_{2_n}(t) - \dot{x}_{2_{n+1}}(t) = \\ \qquad = u_n - f_{n+1}(x_{1_{n+1}}, x_{1_n}, x_{2_{n+1}}, x_{2_n}) \end{cases} \quad (72)$$

Under the hypothesis that $\dot{x}_{2_n}(t) = 0$ and neglecting the friction, we compute the equilibrium points Δx_e in $\Delta v - \Delta d$ plane imposing

$$\begin{cases} \Delta\dot{x}_1(t) = 0 \\ \Delta\dot{x}_2(t) = 0 \end{cases} . \quad (73)$$

Considering the hybrid model defined in (II-A)-(II-D) and imposing the conditions just described, we obtain that the equilibrium points are

$$\Delta x_e = \begin{pmatrix} \Delta x^* \\ 0 \end{pmatrix} \quad (74)$$

with $\Delta x^* \in (\Delta R, +\infty)$, see gray solid line in Figure 5.

Theorem 2: Given the hybrid automaton defined in (II-A)-(II-D), let $S = \{x \in \mathbb{R}^2 \mid (x_1, x_2) \in (\Delta x^*, 0),\ \Delta x^* \in (\Delta R, +\infty)\}$, if the velocity of the leader is constant, i.e. $\dot{x}_{2_n} = 0$, $\forall\ (x, q) \in Init$, $\exists\, T > 0$ such that $x(T) \in S$.

Proof: The sketch of the proof is based on the fact that for any initial state $(\Delta v(t_0), \Delta x(t_0))$, the trajectory of the state variables will arrive in S in a finite time T after a finite number of discrete transitions. The time T is given by the sum of the finite times in which the state remains in every discrete state. In Figure 5 it is depicted a qualitative behavior of the trajectories in each discrete state. For example, if $q(t_0) \in q_7$ and

$$\Delta v(t_0) = x_{2_n}(t_0) - x_{2_{n+1}}(t_0) < 0 \quad (75)$$

$$\Delta x(t_0) = x_{1n}(t_0) - x_{1_{n+1}}(t_0), \quad (76)$$

from (17) it is possible to compute the state variable evolution under the assumtpion that the velocity leader is constant. In particular, if $t_0 = 0$, $\exists\, \bar{t}$, $\bar{\bar{t}}$ such that:

$$\begin{cases} \Delta v(\bar{t}) &= 0 \\ \Delta x(\bar{t}) &= \Delta x(0) + \Delta v(0) \cdot \left(-\frac{\Delta v(0)}{-u_{min}}\right) + \\ &\quad + \frac{1}{2} \cdot (-u_{min}) \cdot \left[\frac{\Delta v^2(0)}{(-u_{min})^2}\right] \\ \bar{t} &= -\frac{\Delta v(0)}{-u_{min}} \end{cases} \quad (77)$$

$$\begin{cases} \Delta x(\bar{\bar{t}}) &= \Delta R \\ \Delta v(\bar{\bar{t}}) &= \Delta v(0) - u_{min} \cdot \bar{\bar{t}} \\ \bar{\bar{t}} &= \frac{-\Delta v(0) + \sqrt{\Delta v^2(0) - 2(-u_{min}) \cdot [\Delta x(0) - \Delta R]}}{f_r - u_{min}} \end{cases} \quad (78)$$

where $\bar{t}$ is the time in which $\Delta v = 0$ and $\bar{\bar{t}}$ is the time to arrive in $\Delta x = \Delta R$ (to move from q_7 to q_5).

Using the same procedure, if $x(t_0) \in q_5$ and $\Delta v(t_0) < 0$, if $t_0 = 0$, $\exists\, t^*$ such that:

$$\begin{cases} \Delta v(t^*) &= 0 \\ \Delta x(t^*) &= \Delta x(0) + \Delta v(0) \cdot (-\Delta v(0) - t_0) + \\ &\quad + \frac{1}{2} \cdot f_r \cdot \left[\frac{\Delta v^2(0)}{f_r^2} - t_0^2\right] \\ t^* &= -\frac{\Delta v(0)}{f_r} \end{cases} \quad (79)$$

If $q(t_0) \in q_5$ and $\Delta v(t_0) > 0$, if $t_0 = 0$, $\exists\, t^{**} \in \mathbb{R}$, that is the time to move from q_5 to q_1, equal to

$$t^{**} = min\{t_1, t_2, t_3\} \quad (80)$$

where

$$t_1 = \frac{f_r\left[-T_s + \sqrt{T_s^2 + \frac{2}{f_r}\left(\bar{k} - \bar{s}_n\right)}\right] - \Delta v(0)}{f_r}, \quad (81)$$

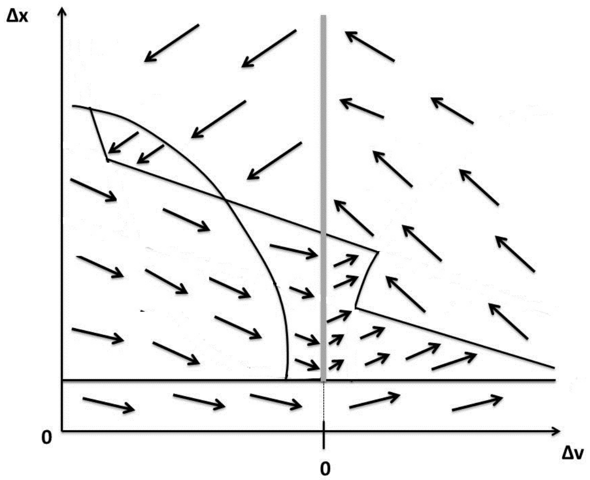

Fig. 5. Equilibrium points set (gray solid line) and qualitative phase portrait.

$$t_2 = \frac{f_r\left[-T_d + \sqrt{T_d^2 + \frac{2}{f_r}\left(\bar{\bar{k}} - \bar{\bar{s}}_n\right)}\right] - \Delta v\left(0\right)}{f_r}, \quad (82)$$

$$t_3 = \frac{2s_nK_{PTP} \pm \sqrt{4s_n^2K_{PTP}^2 - 4\left(f_x - \bar{k}\right)\left(K_{PTP} - \frac{1}{2f_r}\right)}}{2\left(K_{PTP} - \frac{1}{2f_r}\right)}, \quad (83)$$

$$\bar{k} = \Delta x\left(0\right) - \frac{\Delta v^2\left(0\right)}{f_r} + \frac{\Delta v^2\left(0\right)}{2f_r}, \quad (84)$$

$$\bar{s}_n = s_n + T_s x_{2_n},\ \bar{\bar{s}}_n = s_n + T_d x_{2_n}. \quad (85)$$

In this case the time needed to move from q_5 to q_1 is the minimum time for the state trajectory to intersects the lines ΔD, ΔR or the parable PTP according to the defined hybrid system domains. The same procedure can be applied to the remaining cases, $\forall\, q_i \in Q$, it exists finite time $\widehat{t}$ to arrive in S or to move in a new discrete time. ∎

III. Interaction between macroscopic parameters and microscopic model

In this section we assume that $n+1$ vehicle (named as follower) knows not only position and speed of its leader (n vehicle) and of all those vehicles ($n-1$, $n-2$, ...) that precede it on a given spatial interval (M meters). By using these data we calculate macroscopic mean speed and variance values in the same range.

A. Variance-driven time headways model

In [17] authors formulate a variance-driven time headways (VDT) model in terms of a meta-model to be applied to any car-following model where a time headway T_0 for equilibrium traffic can be expressed by a model parameter or a combination of model parameters. They relate on the driver acting not only to his own leader, but also to the neighboring environment.

Starting from the time headway T_0, they obtain a multiplication factor α_T that increases monotonously and is restricted to a maximum value. For doing this, the local velocity average $\bar{v}_N$ and the local variance θ_N, where N is the number of considered cars, are taken into account:

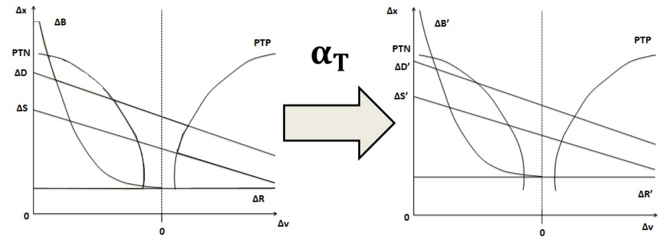

Fig. 6. The different thresholds and regimes in Fritzsche car-following model before and after applying VDT model.

$$\bar{v}_N = \frac{1}{N}\sum_{i=0}^{N-1} x_{2_{n+1-i}} \quad (86)$$

$$\theta_N = \frac{1}{N-1}\sum_{i=0}^{N-1}\left(x_{2_{n+1-i}} - \bar{v}_N\right)^2 \quad (87)$$

Using these parameters the local variation coefficient V_N is defined

$$V_N = \frac{\sqrt[2]{\theta_N}}{\bar{v}_N} \quad (88)$$

such that it is possible to define the multiplication factor α_T in the following way

$$T = \alpha_T T_0 = [\min(1 + \gamma V_N, \alpha_T^{max})]T_0 \quad (89)$$

where γ is the sensitivity of the time headway to increasing velocity variations and α_T^{max} is the maximum value of the multiplication factor. These parameters can be determined from empirical data of the time-headway distribution for free and congested traffic (see [17]).

B. Hybrid model integration with VDT

On the basis of the VDT model, it is possible to include macroscopic parameters as average speed and variance into the microscopic hybrid model defined in Section II changing thresholds definitions. Let us consider the same previous state-dependent guard conditions using the macroscopic information and updating the thresholds values as follows:

$$\Delta R' = s_n + \alpha_T T_r x_{2_n} \quad (90)$$

$$\Delta S' = s_n + \alpha_T T_s x_{2_{n+1}} \quad (91)$$

$$\Delta D' = s_n + \alpha_T T_d x_{2_{n+1}} \quad (92)$$

$$\Delta B' = s_n + \alpha_T T_r x_{2_n} + \frac{(\Delta x_2)^2}{|b_{min}| + a_n^-} \quad (93)$$

Figure 6 shows what happens into the speed-position plane. Now model takes into account parts of the environmental information. In this study we consider the N vehicles in a space of M meters in front of the $n+1$ vehicle for taking into account their speed and consequently the calculated local mean speed and variance. Input data could come from each vehicles or an existing road architecture that measures macroscopic variables in some not specified way. The unique mandatory required microscopic data are about x_n vehicle (see Figure 7).

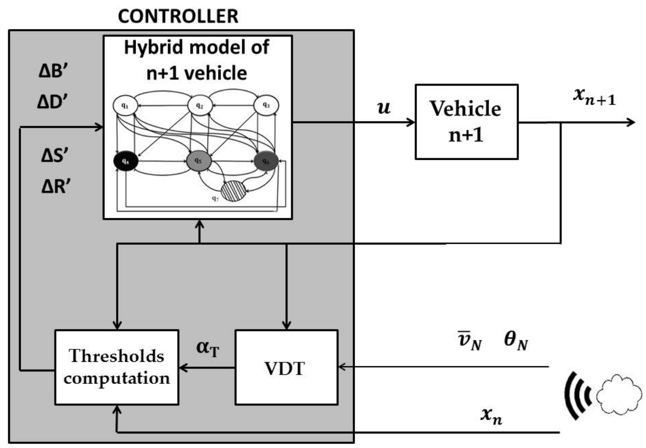

Fig. 7. The adopted architecture. Input data can be either macroscopic ($\bar{v}_N$, θ_N) or microscopic (x_n, x_{n-1}, x_{n-2},...): in the latter case all information are used to compute local mean speed and local variance.

TABLE I
DATA

Parameter	Value	Parameter	Value
L_n [m]	4.5	T_s	1
u_{min}	-7	T_d	1.8
u_{max}	5	α_1^+	0.3
f_r	0.15	α_3^-	-0.2
k_{PTN}	0.002	α_4^-	-1
k_{PTP}	0.002	Δt	0.01
s_n	5.5	γ	4
f_x	0.05	α_T^{max}	2.2
T_r	0.5	M	450

IV. Simulation Results

In this section, we provide simulation results of the proposed control technique. We consider two simulation sets. First we compare the basic controller and one which uses the VDT mechanism when the desired speed is constant showing the improvements if macroscopic data are used. Second we outline simulation results when vehicle platoon varies its speed. For parameters γ and α_T^{max} we adopted the values that had been given from empirical data (cf. Table I). Obviously these parameters would change depending on the considered road.

Safety distance, fuel consumption and emission rate (MOE_e) are the performance indexes considered to quantify differences between controlled systems behaviors. We use MOE_e model which depends on instantaneous speed (s) and acceleration (a):

$$ln(MOE_e) = \begin{cases} \sum_{i=0}^{3}\sum_{j=0}^{3}(L_{i,j}^e \times s^i \times a^j) & for\ a \geq 0 \\ \sum_{i=0}^{3}\sum_{j=0}^{3}(M_{i,j}^e \times s^i \times a^j) & for\ a < 0 \end{cases} \tag{94}$$

Values of $L_{i,j}^e$ and $M_{i,j}^e$ parameters can be found in [18]. In this study we consider 5 vehicles on the segment road of 450 meters in front of the considered vehicle.

TABLE II
CONTINUOUS INITIAL CONDITIONS: CASE I

Vehicle	$x_1(0)$ [m]	$x_2(0)$ [m/s]
n+1	0	30
n	500	25
n-1	600	25
n-2	700	25
n-3	800	25

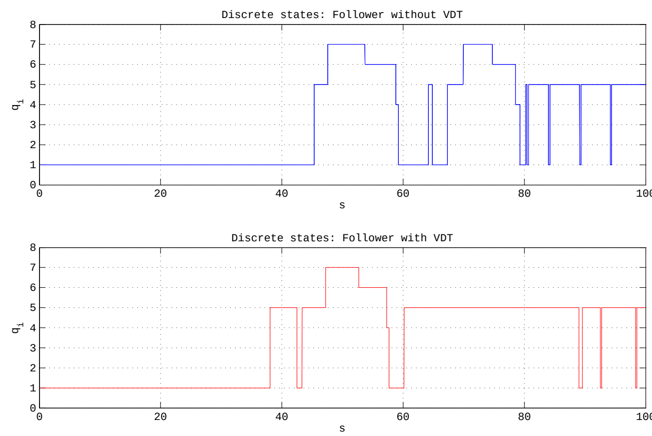

Fig. 8. Case I: Discrete states evolution.

A. Case study I: Platoon approach

Starting condition considered in this case are described by Table II. Vehicle $n+1$ accelerates trying to reach its desired speed of 36 m/s, but it has to decelerate because of the platoon and to tag along to it. Depending on the use of VDT mechanism different discrete evolutions take place, as described by Figure 8. As showed by Figure 9, also continuous behaviors are different: by using VDT system presents a better behavior. Furthermore, minimum distance is bigger in VDT case. According to fuel consumption and emission rate model an improvement of 3.2% is present too: it means that only knowing additional information we can improve vehicle performance. Indeed final results about

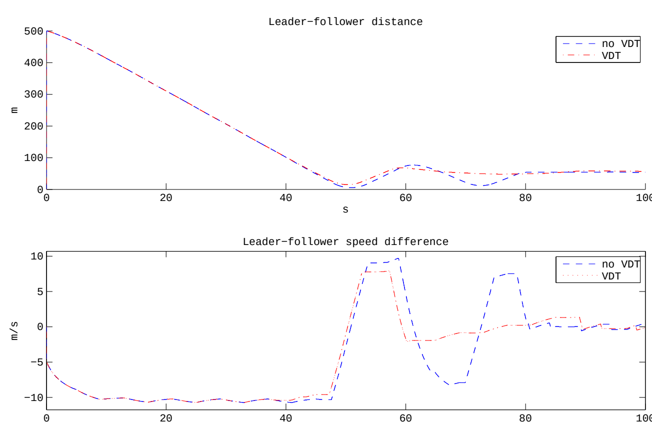

Fig. 9. Case I: Leader-follower interaction.

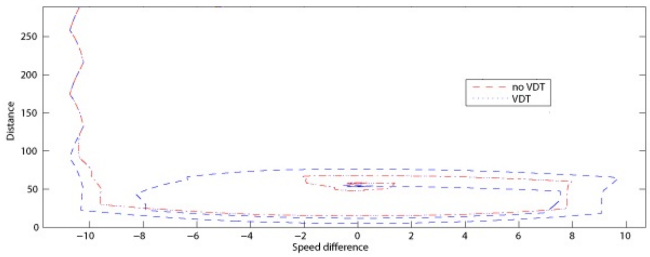

Fig. 10. Case I: Motion described in a speed difference-distance plane.

TABLE III
CONTINUOUS INITIAL CONDITIONS: CASE II

Vehicle	$x_1(0)$ **[m]**	$x_2(0)$ **[m/s]**
n+1	0	22
n	400	22
n-1	500	22
n-2	600	22
n-3	700	22

vehicle state are equal.

B. Case study II: Platoon tracking

Vehicle $n+1$ accelerates trying to reach its desired speed of 30 m/s: after it does, it has to decelerate because of a platoon. Two possible platoon behavior are considered. In the first one, let us intend a string stable platoon without disturbance backward propagation; each vehicle holds the same shifted speed profile (see Figure 12). In the second case, we consider a platoon with speed disturbance propagation occurring on $n-1$ and $n-2$ vehicles. In order to make a good comparison, we suppose that the n vehicle behaves as in the no disturbance case: then the platoon could not be string stable anymore. We compare the use of VDT mechanism in these situations. From Figure 11 we can see that in the disturbance propagation case there is more information respect to one without VDT mechanism. Speed platoon profile is depicted in Figure 13. Because of the different vehicles with time-varying speed there will be an increase of car accident probability. Furthermore because of the disturbance collision probability will still augment. According to this, $n+1$ vehicle driver will keep a higher distance respect to the other case: the higher the probability VDT mechanism faces this situation in a better way because it keeps a bigger distance. collision increase the higher the distance augment. Figure 12 shows each vehicle speed profile in both cases. From Figure 13 it is possible to see that VDT mechanism faces the increase of car accident probability incrementing the distance. Hence the main advantage is given by an improved safety, thanks to augmented distance, and a non increased fuel consumption.

V. CONCLUSIONS

The hybrid system presented in this paper describes different ways a car driver behaves. The automaton is able to introduce dynamical changes depending on next vehicle microscopic behavior and on macroscopic quantities too.

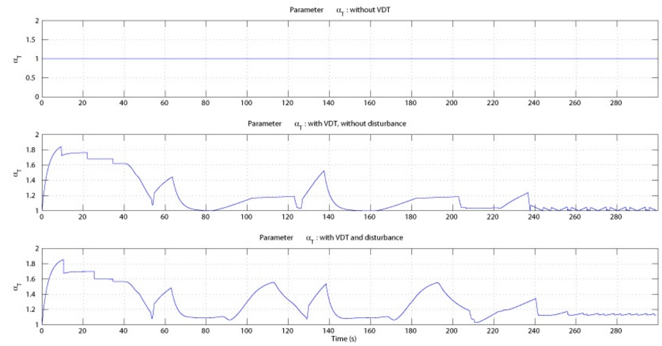

Fig. 11. Case II: α_T profiles in all different situations.

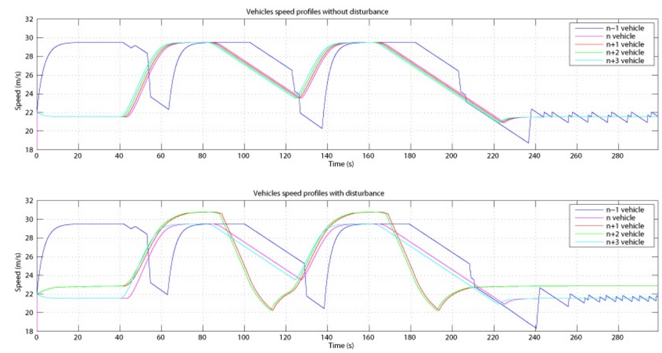

Fig. 12. Case II: Vehicles speed profiles.

Furthermore it similarly behaves as an human driver thanks to psycho-physical model. In an Automatic Cruise Control perspective it performs better from a comfort point of view.

Results described in Section IV show multiple improvement. In fact they cover stuff as interaction distance or fuel consumption or more efficient system behavior.

Given the characteristic to take into account the different aspects related to the real behavior of a driver (comfort desired, acceptable risk of collision, etc.), the proposed model is suitable for being used in two ways: it can be used either as a driving support, which recommends a skillful performance to an human driver, either as a automatic control able to lead the progress as if a human being was driving.

Future work will extend this model for lane-change case. An interesting possibility would also be to take into account

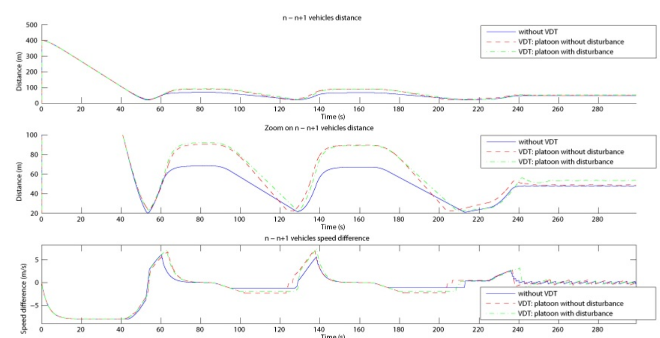

Fig. 13. Case II: Interaction between n and $n+1$ vehicles.

more macroscopic parameters. The focus could also be to try to produce a kind of "positive" shock wave for reducing (or totally neglecting) shock waves propagation.

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