On data-driven controller synthesis with regular language specifications

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21st IFAC World Congress, Germany, July 12-17, 2020

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Introduction

Growing interest from diverse research areas on data-driven techniques for monitoring and controlling complex systems

Our contribution:

- Data–driven control design of abstract systems with regular language specifications
- Abstract systems with unknown structure and no identification
- Information pattern: finite set of experiments collected off–line on the plant
- Results on maximality, convergence and adaptivity of the controller as the set of experiments grows

Abstract systems (1/2)

Abstract systems are collections of input–state functions

- $\mathcal{T} = \{(T_1, T_2) \in \mathbb{N} \times \mathbb{N} | T_1 < T_2\}$ time set
- $\mathcal{U}$ set of input values
- $\mathcal{U}^{[T_1;T_2]}$ set of input functions $u : [T_1; T_2] \rightarrow \mathcal{U}$, with $(T_1, T_2) \in \mathcal{T}$
- $\mathcal{X}$ set of state values
- $\mathcal{X}^{[T_1;T_2]}$ set of state functions $x : [T_1; T_2] \rightarrow \mathcal{X}$, with $(T_1, T_2) \in \mathcal{T}$

Abstract systems (2/2)

An abstract system P is a relation

$$P \subseteq \bigcup_{(T_1,T_2)\in\mathcal{T}} \left(\mathcal{U}^{[T_1;T_2-1]} \times \mathcal{X}^{[T_1;T_2]}\right) \qquad (1)$$

satisfying the following assumptions:

- suffix closure of input-state functions
- concatenation closure of input-state functions
- causality
- time invariance
- determinism
- set of states $\mathcal{X}$ endowed with a metric $\mathbf{d} : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_0^+$

We denote by $\mathcal{X}_0$ the set of initial states of P, i.e.

$$\mathcal{X}_0 = \{x_0 \in \mathcal{X} \mid \exists (u,x) \in P \text{ s.t. } x_0 = x(0)\} \qquad (2)$$

Transition systems (1/2)

A transition system is a tuple

$$S = (X, X_0, U, \longrightarrow, X_m, Y, H) \tag{3}$$

consisting of

- a set of states X
- a set of initial states $X_0 \subseteq X$
- a set of inputs U
- a transition relation $\longrightarrow \subseteq X \times U \times X$
- a set of marked states $X_m \subseteq X$
- a set of outputs Y
- an output function $H : X \rightarrow Y$

A transition $(x, u, x') \in \longrightarrow$ of S is denoted by

$$x \xrightarrow{u} x' \tag{4}$$

Transition systems (2/2)

Given a transition system $S = (X, X_0, U, \longrightarrow, X_m, Y, H)$

- a state run of S is a sequence of transitions

$$x_0 \xrightarrow{u_0} x_1 \xrightarrow{u_1} \dots \xrightarrow{u_{l-1}} x_l \qquad (5)$$

  in S with $x_0 \in X_0$
- S is symbolic if X and U are finite sets
- S is metric if Y is equipped with a metric

Transition systems used in the sequel to model abstract systems, controllers and specifications

Controller and controlled plant P^C

- The controller C is in the form of a transition system

$$C = (X_c, X_{c,0}, U_c, \xrightarrow[c]{}, X_{c,m}, Y_c, H_c) \quad (6)$$

- The plant P controlled by C, denoted by P^C, is described by the collection of pairs $(u, x) \in P$, where control input

$$u \in \mathcal{U}^{[0;T-1]} \quad (7)$$

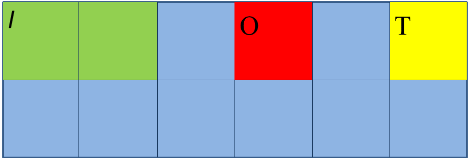
is such that $u(t) = u_t, t \in [0; T-1]$, and sequence

$$\{u_t\}_{t \in [0;T-1]} \quad (8)$$

is the collection of input labels in a state run of C

Specification

- Specification Q is a regular language defined over an alphabet $\mathcal{A}$ that is a finite subset of the set of states $\mathcal{X}$ of the plant P
- Recall: A language L over $\mathcal{A}$ is a collection of sequences with finite length (called words or strings) with symbols in $\mathcal{A}$. L is regular if it can be represented by a symbolic transition system
- Example:
  Q = Starting from I reach T while avoiding O



Q = Regular language collecting all words starting with green symbols, ending with yellow symbols and with no red symbols

Control problem formulation

- Consider an abstract system P, a finite set of experiments $\mathcal{E} \subseteq P$ and a desired accuracy $\theta \geq 0$
- Find a controller C and a relation $\mathcal{R}_0 \subseteq \mathcal{X}_0 \times X_{c,0}$, which enforce specification Q on P up to accuracy θ, i.e. such that for any pairs

$$(u, x) \in P^C \cap (\mathcal{U}^{[0;T-1]} \times \mathcal{X}^{[0;T]}), \tag{9}$$

for some $(0, T) \in \mathcal{T}$, with $(x(0), x_{c,0}) \in \mathcal{R}_0$, there exists a word $q_0\, q_1 \ldots q_T \in Q$ such that:

$$\mathbf{d}(x(t), q_t) \leq \theta, \; \forall t \in [0; T] \tag{10}$$

where we recall $\mathbf{d}$ is the metric endowed on $\mathcal{X}$

Solution scheme

- (Step 1) construction of a symbolic transition system $S(\mathcal{E})$ encoding the set of experiments $\mathcal{E}$
- (Step 2) construction of a symbolic transition system S_Q encoding the specification Q
- (Step 3) computation of the approximate parallel composition

$$C' = S(\mathcal{E}) \times_\theta S_Q \tag{11}$$

  between $S(\mathcal{E})$ and S_Q with accuracy $\theta \geq 0$
- (Step 4) computation of the controller C given as the accessible and co-accessible part of C', i.e.

$$C = Trim(C') \tag{12}$$

- (Step 5) derivation of the relation $\mathcal{R}_0 \subseteq \mathcal{X}_0 \times X_{c,0}$

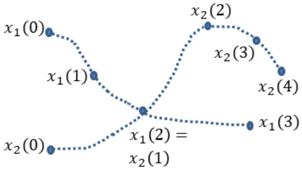
Step 1

Symbolic transition system encoding the set of experiments $\mathcal{E}$ is

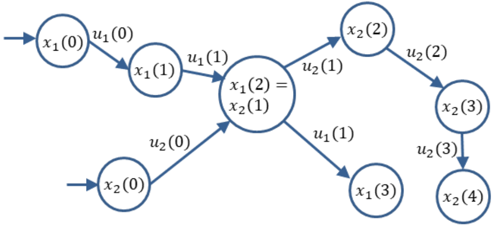
$$S(\mathcal{E}) = (X_e, X_{e,0}, U_e, \xrightarrow[e]{}, X_{e,m}, Y_e, H_e) \tag{13}$$

Example:

$\mathcal{E} = \{(u_1, x_1), (u_2, x_2)\} =$



$S(\mathcal{E}) =$



Step 1

Symbolic transition system encoding the set of experiments $\mathcal{E}$ is

$$S(\mathcal{E}) = (X_e, X_{e,0}, U_e, \xrightarrow[e]{}, X_{e,m}, Y_e, H_e), \tag{14}$$

where

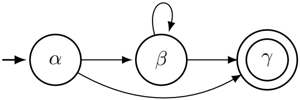
- $X_e = \{z \in \mathcal{X} \mid \exists (u,x) \in \mathcal{E} \land \exists t \in \mathbb{N} \text{ s.t. } z = x(t)\}$
- $X_{e,0} = X_e \cap \mathcal{X}_0$
- $U_e = \mathcal{U}$
- $z \xrightarrow[e]{v} z^+$, if there exist $(u,x) \in \mathcal{E}$ and $t \in \mathbb{N}$ such that $z = x(t)$, $v = u(t)$ and $z^+ = x(t+1)$
- $X_{e,m} = X_e$
- $Y_e = \mathcal{X}$
- $H_e(x) = x$ for any $x \in X_e$

Step 2

Symbolic transition system $S_Q = (X_Q, X_{Q,0}, U_Q, \xrightarrow[Q]{}, X_{Q,m}, \mathcal{A}, H_Q)$ encoding the specification Q can be obtained by using standard results on language theory

Example:

- Consider Q requiring to reach a symbol γ starting from a symbol α while only symbols β are allowed
- Corresponding regular language is $\{\alpha\}\{\beta\}^*\{\gamma\}$ where $\{\beta\}^*$ is the Kleene closure of β, i.e. the collection of all words with finite length and with symbols β
- S_Q encoding Q is a symbolic transition system



Step 3

$$C' = S(\mathcal{E}) \times_{\theta} S_Q = (X'_c, X'_{c,0}, U'_c, \xrightarrow[c,l]{}, X'_{c,m}, \mathcal{X}, H'_c), \qquad (15)$$

where:

- X'_c is the collection of pairs $(x_e, x_Q) \in X_e \times X_Q$ such that

$$\mathbf{d}(H_e(x_e), H_Q(x_Q)) \leq \theta \qquad (16)$$

- $X'_{c,0} = X'_c \cap (X_{e,0} \times X_{Q,0})$
- $U'_c = \{v \in \mathcal{U} \,|\, \exists (u,x) \in \mathcal{E} \wedge \exists t \in \mathbb{N} \text{ s.t. } v = u(t)\}$
- $(x_e, x_Q) \xrightarrow[c,l]{u} (x_e^+, x_Q^+)$ if $x_e \xrightarrow[e]{u} x_e^+$ and $x_Q \xrightarrow[Q]{} x_Q^+$
- $X_{c,m} = X_e \times X_{Q,m}$
- $H'_c(x_e, x_Q) = H_Q(x_Q)$ for any $(x_e, x_Q) \in X'_c$

Steps 4 and 5

- Construct the controller C as the accessible and co-accessible part of C', i.e.

$$C = Trim(C') \tag{17}$$

- Define relation $\mathcal{R}_0$ by

$$\mathcal{R}_0 = \{(x, (x_e, x_Q)) \in \mathcal{X}_0 \times X_{c,0} | x = x_e\} \tag{18}$$

Theorem

Controller C and relation $\mathcal{R}_0$ solve our control problem

Additional results

- (Maximality) There is no controller other than C which can enforce a larger part of the specification Q on the plant P
- (Convergence properties) Consider a sequence of finite sets of experiments $\mathcal{E}_i \subseteq \mathcal{E}_{i+1} \subseteq P$, $i \in \mathbb{N}$. Then, there exists $i^* \in \mathbb{N}$ such that for any $i \geq i^*$ the controlled plant P^C cannot achieve a part of the specification Q which is larger than the one obtained at step i^*
- (Adaptivity) A controller to be designed on the basis of a set of experiments $\mathcal{E}_1 \cup \mathcal{E}_2$ can be obtained by computing the "union" of the controllers C_1 and C_2 designed independently for the sets of experiments $\mathcal{E}_1$ and $\mathcal{E}_2$, respectively

Application to the Artificial Pancreas (1/2)

Plant

$$P : \begin{cases} \frac{dG(\tau)}{d\tau} = -K_{xgi} G(\tau) I(\tau) + \frac{T_{gh}}{V_G}, \\ \frac{dI(\tau)}{d\tau} = -K_{xi} I(\tau) + \frac{T_{iGmax}}{V_I} h(G(\tau)) + u(\tau) \end{cases} \quad (19)$$

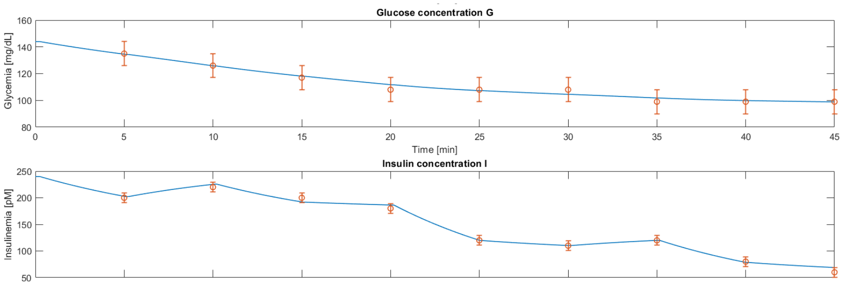
where

- G is the glucose concentration in the plasma
- I is the insulin concentration in the plasma
- u is the exogenous intra-venous Insulin Delivery Rate (IDR)
- h represents the endogenous pancreatic IDR and defined as

$$h(G) = \frac{(G/G^*)^\gamma}{1 + (G/G^*)^\gamma} \quad (20)$$

Control objective: Drive state variables in safe regions

Application to the Artificial Pancreas (2/2)



- 20,000 experiments collected by simulating the model of the plant P that is however assumed to be unknown
- Time of computation 1,852 sec on a laptop with CPU Intel Core i7-6700HQ at 2.60 GHz

Conclusions

- Data–driven control design of abstract systems with specifications expressed in terms of regular languages
- Maximality of the controller, convergence of the controller as the number of experiments increases and adaptive–type control design
- Preliminary results on the application to the Artificial Pancreas