Modelling, Analysis and Design of Wireless Networked Control Systems

Alessandro D’Innocenzo

Department of Information Engineering, Computer Science and Mathematics
Center of Excellence for Research DEWS, University of L’Aquila
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Challenge: close the loop around wireless multi-hop control networks.
Wireless control systems

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WirelessHART MAC layer (scheduling)

- time is divided in periodic frames, each divided in $\Pi$ time slots, each of duration $\Delta$
- to avoid interference, a periodic scheduling allows each node to transmit data only in a subset of time slots
- model impact of scheduling on the closed-loop dynamics

![Diagram of WirelessHART MAC layer](image-url)
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WirelessHART network layer (routing)

- Single path vs multi path routing
- Static vs Dynamic routing
- Redundancy in the data routing (flooding) and network coding
Wireless control networks as switching systems

**Mathematical model:** \( x(t + 1) = A(\sigma(t))x(t) + B(\sigma(t))u(t), t \in \mathbb{N} \), where \( x(t) \) is the plant state, \( \sigma(t) \in \Sigma \) depends on routing/scheduling and \( u(t) = K(t)x(t) \) is the control signal. The communication parameters are considered as a disturbance.
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Problem 1: given $K(t)$, verify whether the closed loop systems is asymptotically stable, i.e. the Joint Spectral Radius of $\{A(\sigma(t)) + B(\sigma(t))K(t)\}_{\sigma(t) \in \Sigma}$ is smaller than 1.

Problem 2: Design a controller $K(t)$ such that the closed loop system is asymptotically stable.

Insight: stabilizability depends on our knowledge of the switching signal $\sigma(t)$:

- We cannot measure $\sigma(t)$: then $K(t) = K, \forall t \in \mathbb{N}$
- We can measure and keep memory of $\sigma(t), K(t) = K\left(\sigma(t - d) \cdots \sigma(t)\right)$
- We also have a finite horizon knowledge of future $N$ switching signals $\sigma(t): K(t) = K\left(\sigma(t - d) \cdots \sigma(t + N)\right)$
Wireless control networks as switching systems

Collaborations:
Raphael Jungers (Université Catholique de Louvain)
Nicola Guglielmi (University of L'Aquila)
George Pappas (University of Pennsylvania)

Selected Publications:

Fault tolerant stabilizability of wireless control networks

Mathematical model:
\[ x(t + 1) = Ax(t) + Bu(t) \]
\[ y = Cx(t), \ t \in \mathbb{N} \]
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**Assumption:** Failures are slow with respect to plant time constants
Fault tolerant stabilizability of wireless control networks

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**Problem 1:** *Design* the communication system parameters of \( G_R \) and \( G_O \) (topology, scheduling, routing) to guarantee existence of a stabilizing controller for any failure

**Problem 2:** *Design* the communication system parameters to guarantee the existence of a Fault Detection and Isolation (FDI) systems

**Insight:** Translate classical stabilizability and FDI conditions on LTI systems to conditions on the communication system parameters
Fault tolerant stabilizability of wireless control networks

Selected Publications:


Co-analysis and co-design of wireless control systems using finite probabilistic abstractions

Mathematical model: $S: x(t + 1) = f(x(t), \sigma(t), u(t), p(t)), t \in \mathbb{N}$:

- $x(t)$ is the plant state, $f(\cdot)$ is a non-linear function
- $\sigma(t) \in \Sigma$ is a Markov Chain depending on routing, scheduling, and packet losses
- $u(t)$ is the plant control signal
- $p(t) \in P$ is the transmission power control signal
Co-analysis and co-design of wireless control systems using finite probabilistic abstractions

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**Problem 1:** given the control signals $u(t), p(t)$, verify whether the closed loop system satisfies a probabilistic property (e.g. unsafe with probability $< 10^{-9}$)

**Insight:** Derive a **Markov Chain** abstraction of the controlled stochastic process $S$ with precision $\varepsilon$ and use **Model Checking** techniques. Need models to handle both non-determinism and stochasticity: **Markov-set chains, Interval Markov Chains**.

**Problem 2:** given $S$, design a control laws for $u(t), p(t)$ such that the closed loop system satisfies a probabilistic property.

**Insight:** Derive a **Markov Decision Process** (MDP) abstraction of the stochastic process $S$ with precision $\varepsilon$ and use design techniques for MDPs.
Wireless control networks as switching systems

Collaborations:
Alessandro Abate (University of Oxford)
Joost-Pieter Katoen (RWTH Aachen University)
Claudia Rinaldi and Fortunato Santucci (University of L'Aquila)

Selected Publications:
